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I.Sc. XII Relations and Functions. Date-05-05-2020

Q.1. Determine whether each of the following relations are reflexive, symmetric and transitive.

(i) Relation R in set A of human beings in a town at a particular time given by
~~(d) R = {(x, y): x is wife of y}~~
(e) R = {(x, y): x is father of y}.

Soln: It is clear that R is not reflexive as x cannot be father of x. For any x, y is not symmetric as if x is father of y, then y cannot be father of x. R is not transitive as if x is father of y and y is father of z, then x is not father of z.

Q.2. Show that the relation R in the set R of real numbers, defined as $R = \{(a,b): a \leq b^2\}$ is neither reflexive nor symmetric nor transitive.

Soln: We have, $R = \{(a,b): a \leq b^2\}$, where $a, b \in R$

For reflexivity: We observe that $\frac{1}{2} \leq \left(\frac{1}{2}\right)^2$ is not true.

So, R is not reflexive as $\left(\frac{1}{2}, \frac{1}{2}\right) \notin R$

For symmetry: we observe that $-1 \leq 3^2$ but $3 \not\leq (-1)^2$
$\therefore (-1, 3) \in R$ but $(3, -1) \notin R$. So, R is not symmetric.

For transitivity: We observe that $2 \leq (-3)^2$ and $-3 \leq 1^2$ but $2 \not\leq (1)^2$. $\therefore (2, -3) \in R$ and $(-3, 1) \in R$ but $(2, 1) \notin R$. So, R is not transitive.

Hence, R is neither reflexive, nor symmetric and nor transitive.

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Q.3 Check whether the relation R defined in the set $\{1,2,3,4,5,6\}$ as $R = \{(a,b) : b = a+1\}$ is reflexive, symmetric or transitive.

Soln: Let $A = \{1,2,3,4,5,6\}$.

Then, a relation R is defined on set A is $R = \{(a,b) : b = a+1\}$. Therefore, $R = \{(1,2),(2,3),(3,4),(4,5),(5,6)\}$

Now, $6 \in A$ but $(6,6) \notin R$.

Therefore, R is not reflexive.

It can be observed that $(1,2) \in R$ but $(2,1) \notin R$.

Therefore, R is not symmetric.

Now, $(1,2),(2,3) \in R$ but $(1,3) \notin R$.

Therefore, R is not transitive.

Hence, R is neither reflexive, nor symmetric nor transitive.

Q.4. Show that the relation R in R defined as $R = \{(a,b) : a \le b\}$, is reflexive and transitive but not symmetric.

Soln We have, $R = \{(a,b) : a \le b$.

Let $a, b \in R$

For reflexivity: For any $a \in R$ We have $a \le a$.

So, R is reflexive.

For symmetry: We observe that $(2,3) \in R$ but $(3,2) \notin R$.

So, R is not symmetric.

For transitivity: For any $a, b \in R$. We have $(a,b) \in R$ and $(b,c) \in R \Rightarrow a \le b$ and $b \le c \Rightarrow a \le c \Rightarrow (a,c) \in R$.

So, R is transitive. Hence, R is reflexive and transitive but not symmetric. Proved.